

The Hadronic Contribution to $(g - 2)_\mu$

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The evaluation of the hadronic contribution to the muon magnetic anomaly a_μ is revisited, taking advantage of new experimental data on e^+e^- annihilation into hadrons: SND and CMD-2 for the $\pi^+\pi^-$ channel, and *BABAR* for multihadron final states. Discrepancies are observed between KLOE and CMD-2/SND data, preventing one from averaging all the e^+e^- results. The long-standing disagreement between spectral functions obtained from τ decays and e^+e^- annihilation is still present, and not accounted by isospin-breaking corrections, for which new estimates have been presented. The updated Standard Model value for a_μ based on e^+e^- annihilation data is now reaching a precision better than experiment, and it disagrees with the direct measurement from BNL at the 3.3σ level, while the τ -based estimate is in much better agreement. The τ/e^+e^- discrepancy, best revealed when comparing the measured branching fraction for $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ to its prediction from the isospin-breaking-corrected e^+e^- spectral function, remains a serious problem to be understood.

1. Introduction

Hadronic vacuum polarization (HVP) in the photon propagator plays an important role in many precision tests of the Standard Model. This is the case for the muon magnetic anomaly $a_\mu \equiv (g_\mu - 2)/2$, where the HVP component is the leading contributor to the uncertainty of the Standard Model prediction. The HVP contribution is computed by means of a dispersion relation as an integral over experimentally determined spectral functions. It is a property of this dispersion relation that the $\pi\pi$ spectral function provides the major part of the total HVP contribution, so that the experimental effort focuses on this channel.

Spectral functions are directly obtained from the measured cross sections of e^+e^- annihilation into hadrons. The accuracy of the HVP predictions has therefore followed the progress in the quality of the data [1] it relies on. Because the data quality was not always suitable, it was deemed necessary to resort to other sources of information. One such possibility was the use of the vector spectral functions [2] derived from the study of hadronic τ decays [3] for the energy range less than $m_\tau \simeq 1.8 \text{ GeV}/c^2$. For this purpose, the isospin rotation that leads from the

charged τ to the neutral e^+e^- final state has to be thoroughly corrected for isospin-breaking effects.

Also, it was demonstrated that essentially perturbative QCD could be applied to energy scales as low as 1–2 GeV [4, 5], thus offering a way to replace poor e^+e^- data in some energy regions by a reliable and precise theoretical prescription [6, 7, 8, 9, 10, 11].

Detailed reanalyses including all available experimental data have been published in Refs. [12, 13, 14, 15, 16], taking advantage of precise results in the $\pi\pi$ channel from the CMD-2 experiment [17] and from the ALEPH analysis of τ decays [18], and benefiting from a more complete treatment of isospin-breaking corrections [19, 20]. With the increased accuracy of the e^+e^- data a discrepancy with the isospin-breaking-corrected τ spectral functions was found [12], thus leading to inconsistent predictions for the lowest-order hadronic contribution to a_μ . The dominant contribution to this discrepancy stems from the $\pi^+\pi^-$ channel, although another discrepancy occurs in the $\pi^+\pi^-2\pi^0$ mode.

Improvements in the HVP calculation are needed in order to match the present experimental accuracy on a_μ from the BNL experiment [

21],

$$a_\mu = (11\,659\,208.0 \pm 6.3) \times 10^{-10} . \quad (1)$$

In this paper I revisit the input to the HVP dispersion integral in the light of new experimental data on $e^+e^- \rightarrow \pi^+\pi^-$ from SND [22] and CMD-2 [23, 24, 25], and on multihadron final states from *BABAR* [26, 27, 28, 29] using the radiative return technique [30]. These new measurements represent a significant step forward, as they overcome in precision previous determinations in the same channels.

2. Muon Magnetic Anomaly

It is convenient to separate the Standard Model (SM) prediction for the anomalous magnetic moment of the muon into its different contributions,

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}} , \quad (2)$$

with

$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}} , \quad (3)$$

and where $a_\mu^{\text{QED}} = (11\,658\,471.9 \pm 0.1) \times 10^{-10}$ is the pure electromagnetic contribution [31], $a_\mu^{\text{had,LO}}$ is the lowest-order HVP contribution, $a_\mu^{\text{had,HO}} = (-9.8 \pm 0.1) \times 10^{-10}$ is the corresponding higher-order part [32, 2, 14], and $a_\mu^{\text{weak}} = (15.4 \pm 0.1 \pm 0.2) \times 10^{-10}$, where the first error is the hadronic uncertainty and the second is due to the Higgs mass range, accounts for corrections due to exchange of the weakly interacting bosons up to two loops [33]. For the light-by-light (LBL) scattering part, $a_\mu^{\text{had,LBL}}$, we use the value $(12.0 \pm 3.5) \times 10^{-10}$ from the latest evaluation [34, 35], slightly corrected for the missing contribution from (mainly) the pion box.

Owing to unitarity and to the analyticity of the vacuum-polarization function, the lowest order HVP contribution to a_μ can be computed via the dispersion integral [36]

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R^{(0)}(s) , \quad (4)$$

where $K(s)$ is a well-known QED kernel, and $R^{(0)}(s)$ denotes the ratio of the “bare” cross

section for e^+e^- annihilation into hadrons to the pointlike muon-pair cross section. The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, photon radiation in the final state is included in the bare cross section defined here. The reason for using the bare (*i.e.*, lowest order) cross section is that a full treatment of higher orders is anyhow needed at the level of a_μ , so that the use of the “dressed” cross section would entail the risk of double-counting some of the higher-order contributions.

The function $K(s) \sim 1/s$ in Eq. (4) gives a strong weight to the low-energy part of the integral. About 91% of the total contribution to $a_\mu^{\text{had,LO}}$ is accumulated at center-of-mass energies \sqrt{s} below 1.8 GeV and 73% of $a_\mu^{\text{had,LO}}$ is covered by the $\pi\pi$ final state, which is dominated by the $\rho(770)$ resonance.

3. The 2π Input Data

A detailed compilation of all the experimental data used in the evaluation of the dispersion integral (4) prior to 2004 is provided in Refs. [13, 12]. Also discussed therein is the corrective treatment of radiative effects applied to some of the measurements. The τ 2π spectral function is obtained by averaging the results from ALEPH [3], CLEO [37] and OPAL [38], which exhibit satisfactory mutual agreement. Since 2004, new cross section measurements became available from KLOE [39] using radiative return at DAPHNE and from the annihilation experiments SND [22] and CMD-2 [23, 24, 25] at Novosibirsk.

The comparison between all new e^+e^- results and the combined τ spectral function is given in Fig. 1.

A few remarks are in order:

- Revision of the radiative corrections applied the CMD-2 (94-95 data) and SND data led to corrections amounting up to 3%.
- The revised SND and CMD-2 (also including new data released in 2006) spectral

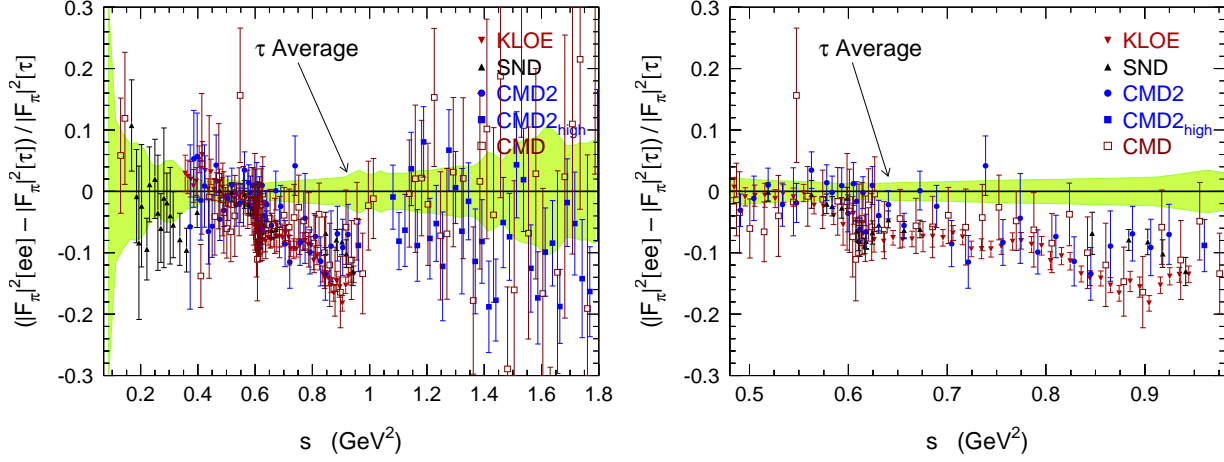


Figure 1. *Relative comparison of the $\pi^+\pi^-$ spectral functions from e^+e^- -annihilation data and isospin-breaking-corrected τ data, expressed as a ratio to the τ spectral function. The shaded band indicates the errors of the τ data. The e^+e^- data are from KLOE [39], CMD-2 [17], CMD, OLYA and DM1 (references given in Ref. [12]). The right hand plot emphasizes the region of the ρ peak.*

functions agree within errors. It should be pointed out that both analyses now use the same radiative correction package, introducing a full correlation between the two data sets.

- The high-statistics KLOE data do not agree with SND and CMD-2, mostly through a discrepancy in the ρ lineshape: KLOE is higher below the peak and becomes lower above.
- A significant discrepancy, most visible above the ρ peak, but present almost everywhere, is found between τ and the e^+e^- data.

Considering different ρ^- and ρ^0 masses as an additional isospin-breaking correction of the τ spectral function would improve the comparison in the ρ resonance peak, but in that case the discrepancy should fade away for masses above $m_\rho + \Gamma_\rho/2$, which is not observed.

Finally, some improvement of the isospin-breaking corrections has been proposed in Ref. [40]. A contribution from the $\rho\omega\pi$ vertex with $\omega \rightarrow \pi^0\gamma$ was not included in the treatment used

so far [20], as it occurs from higher order in Chiral Perturbation Theory. It was however found to be significant, but its effect amounts to only 20% of the observed discrepancy.

4. Testing CVC

Measurement of branching fractions in τ decays are more robust than the spectral functions, as the latter ones depend on the experimental resolution and require a numerically delicate unfolding. It is possible to relate the measured branching ratios for $\tau^- \rightarrow V^- \nu_\tau$, where V is any vector final state, to their expectations from CVC using e^+e^- spectral functions, duly corrected for isospin breaking. In this way we do not anymore rely on the shape of the τ spectral function and instead concentrate on the relative normalization and the isospin-breaking corrections.

The result of the test for the $\pi^-\pi^0$ channel is shown in Fig. 2. It shows a large discrepancy between the average τ branching ratio and the CVC prediction. The difference $[B_\tau - B_{CVC}]_{\pi\pi^0} = (0.92 \pm 0.21)\%$ is 4.5σ away from zero. In relative terms, the discrepancy is a 3.6% effect, about twice the already applied isospin-breaking correction, dominated by (expected to be) well-

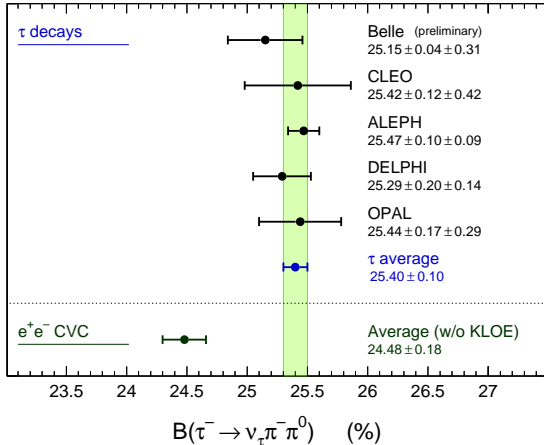


Figure 2. The measured branching ratios for $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \rightarrow \pi^+\pi^-$ spectral function applying the isospin-breaking correction factors discussed in Ref. [13]. The measured branching ratios are from ALEPH [18], CLEO [41] and OPAL [42]. The L3 and OPAL results are obtained from their $h\pi^0$ branching ratio, reduced by the small $K\pi^0$ contribution measured by ALEPH [43] and CLEO [44].

controlled short-distance effects.

The same exercise can be applied to other channels. For the $\pi 3\pi^0$ final state, the difference is consistent with zero, $-(0.08 \pm 0.11)\%$. The result for the $3\pi\pi^0$ mode is bad, $(0.91 \pm 0.25)\%$, amounting to a relative 20% discrepancy, certainly much beyond any reasonable isospin-breaking effect. Here the quality of the data on $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ is in doubt, as they show a considerable spread between the experiments.

5. New Multihadron Data

Results from the BABAR [26, 27, 28, 29] experiment are being produced on the multihadron final states using radiative return [30]. They are part of a program designed to cover most exclusive annihilation processes in the few GeV energy range, taking advantage of the large initial centre-of-mass energy of 10.6 GeV. Hard-radiated pho-

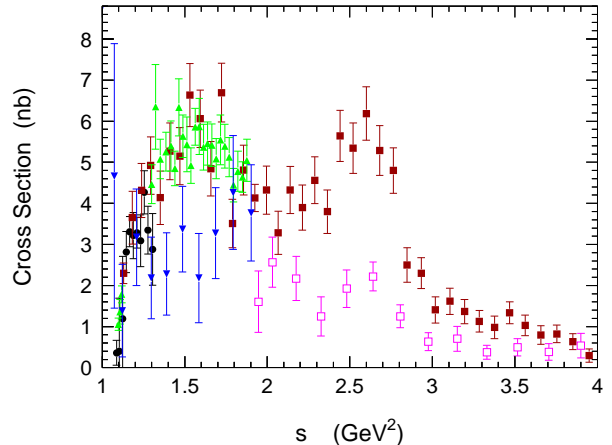


Figure 3. The measured cross section for $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ from BABAR [26] (indicated by full squares) compared to previous measurements (see references in Ref. [13]).

tons are detected at large angle, together with the hadronic system byproducts, so that the full final state can be kinematically constrained (see the discussion in Ref. [29]). These results significantly improve the corresponding contributions to a_μ^{had} , since earlier data in the 1.4-3 GeV energy range were of poor statistical and systematic quality.

The BABAR results are compared to other data in Figs. 3-6 for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, $3\pi^+3\pi^-$, and $2\pi^+2\pi^-2\pi^0$ cross sections. Besides a generally good agreement with previous experiments, some important differences are seen for $\pi^+\pi^-\pi^0$ and the 6-pion states with the results obtained at DCI. The impact of these new measurements is quantified in Table 1.

6. Results

During the previous evaluations of $a_\mu^{\text{had,LO}}$, the results using respectively the τ and e^+e^- data were quoted individually, but on the same footing since the e^+e^- -based evaluation was dominated by the data from a single experiment (CMD-2). The confirmation of the τ/e^+e^- discrepancy by SND and KLOE may suggest to prefer the

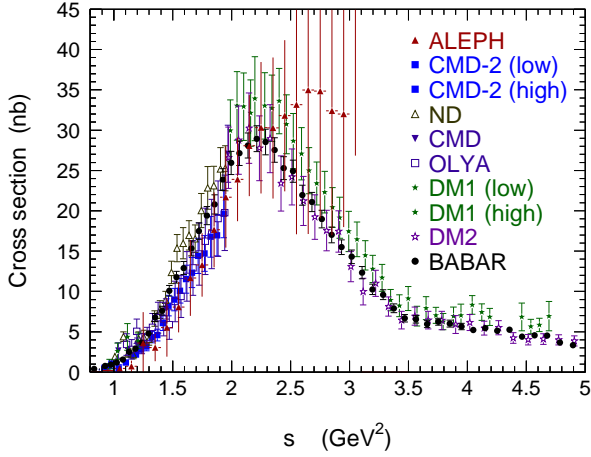


Figure 4. The measured cross section for $e^+e^- \rightarrow 2\pi^+2\pi^-$ from BABAR [27] (indicated by full circles) compared to previous measurements and results from $\tau^- \rightarrow \pi^-3\pi^0$ (see references in Ref. [13]).

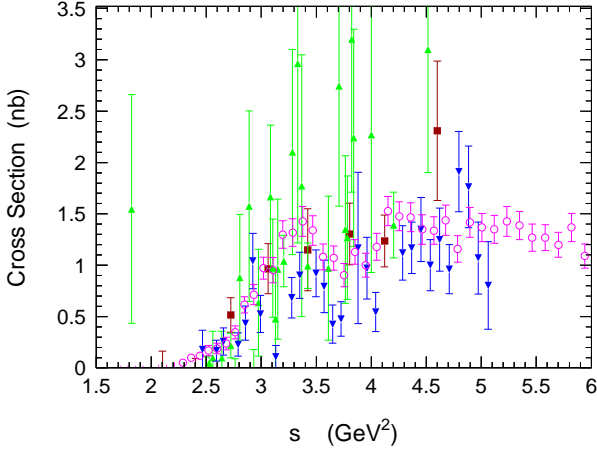


Figure 5. The measured cross section for $e^+e^- \rightarrow 3\pi^+3\pi^-$ from BABAR [28] (indicated by open circles) compared to previous measurements (see references in Ref. [13]).

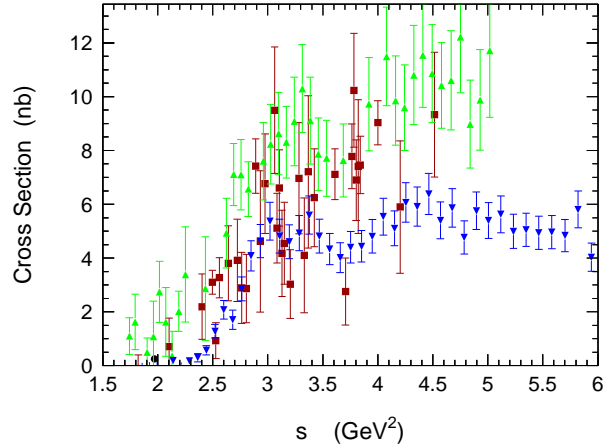


Figure 6. The measured cross section for $e^+e^- \rightarrow 2\pi^+2\pi^-2\pi^0$ from BABAR [28] (indicated by triangles pointing below) compared to previous measurements (see references in Ref. [13]).

Table 1

The contribution of some multipion processes to a_μ^{had} integrated from threshold to 1.8 GeV for the older experiments (references in Ref. [13]) and including the new BABAR ISR results [26, 27, 28, 29]. Values are given in units of 10^{-10} .

process	older exp.	with BABAR
$\pi^+\pi^-\pi^0$	2.45 ± 0.26	3.25 ± 0.09
$2\pi^+2\pi^-$	14.20 ± 0.90	13.09 ± 0.44
$3\pi^+3\pi^-$	0.10 ± 0.10	0.11 ± 0.02
$2\pi^+2\pi^-2\pi^0$	1.42 ± 0.30	0.89 ± 0.09

e^+e^- -based result until a better understanding of the dynamical origin of the observed effect is achieved. This discrepancy is a challenging problem, which may itself turn out to be of fundamental importance. The present update is therefore only based on e^+e^- data, dominated by the latest results from CMD-2 and SND which are in good agreement. The KLOE data show a systematic trend which is not explained at the moment, so we do not use them as the resulting increase in precision would not be trustworthy. Further studies by KLOE are ongoing, in particular a determination of the $\pi\pi/\mu\mu$ ratio where several systematic effects cancel, which should lead a significant improvement of the systematic uncertainty [45].

The preliminary estimate of the integral (4) given below includes one additional improvement with respect to Ref. [12]: perturbative QCD is used instead of experimental data in the region between 1.8 and 3.7 GeV, where non-perturbative contributions to integrals over differently weighed spectral functions were found to be small [7]. This results in a reduction of $a_\mu^{\text{had,LO}}$ by -1×10^{-10} . All contributions to the dispersion integral where no new input data are available are taken from Ref. [12].

The R values from data and QCD are displayed in Fig. 7, but not yet updated with the *BABAR* multipion data. For masses larger than 1.8 GeV, except in the $c\bar{c}$ threshold region from 3.7 to 5 GeV, the QCD prediction is used. Agreement between QCD and data is good. The contributions of the different exclusive channels and of the continuum are given in Table 2.

The e^+e^- -based result for the lowest order hadronic contribution is

$$a_\mu^{\text{had,LO}} = (690.8 \pm 3.9 \pm 1.9_{\text{rad}} \pm 0.7_{\text{QCD}}) \times 10^{-10}, \quad (5)$$

where the second error is due to our treatment of (potentially) missing radiative corrections in the older data [13]. For comparison, the τ -based result [12] can be updated using the new e^+e^- *BABAR* data for the channels other than 2π or 4π yielding the value $(710.3 \pm 5.2) \times 10^{-10}$. Adding to the e^+e^- result (5) the QED, higher-order hadronic, light-by-light scattering, and weak contributions given in Section 2, one finds

$$a_\mu^{\text{SM}} = (11\,659\,180.5 \pm 4.4_{\text{had,LO+HO}})$$

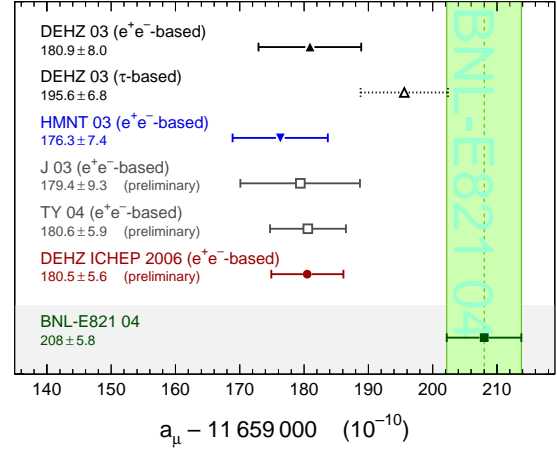


Figure 8. Comparison of the result (6) with the BNL measurement [21]. Also given are our previous estimates [12], where the triangle with the dotted error bar indicates the τ -based result, as well as the estimates from Refs. [14, 15, 16], not yet including the KLOE data.

$$\pm 3.5_{\text{had,LBL}} \pm 0.2_{\text{QED+EW}}) \times 10^{-10}. \quad (6)$$

This value can be compared to the present measurement (1); adding all errors in quadrature, the difference between experiment and theory is

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.5 \pm 8.4) \times 10^{-10}, \quad (7)$$

which corresponds to 3.3 “standard deviations” (to be interpreted with care due to the dominance of systematic errors in the SM prediction). A graphical comparison of the result (6) with previous evaluations¹ and the experimental value is given in Fig. 8.

7. Conclusion and Perspectives

In spite of the new and precise data on the two-pion spectral function from CMD-2 and SND, and on multihadron cross sections from *BABAR*, the lowest order hadronic vacuum-polarization contribution remains the most critical component in the Standard Model prediction of a_μ . Yet, for

¹Results similar to ours have just appeared in Ref. [46].

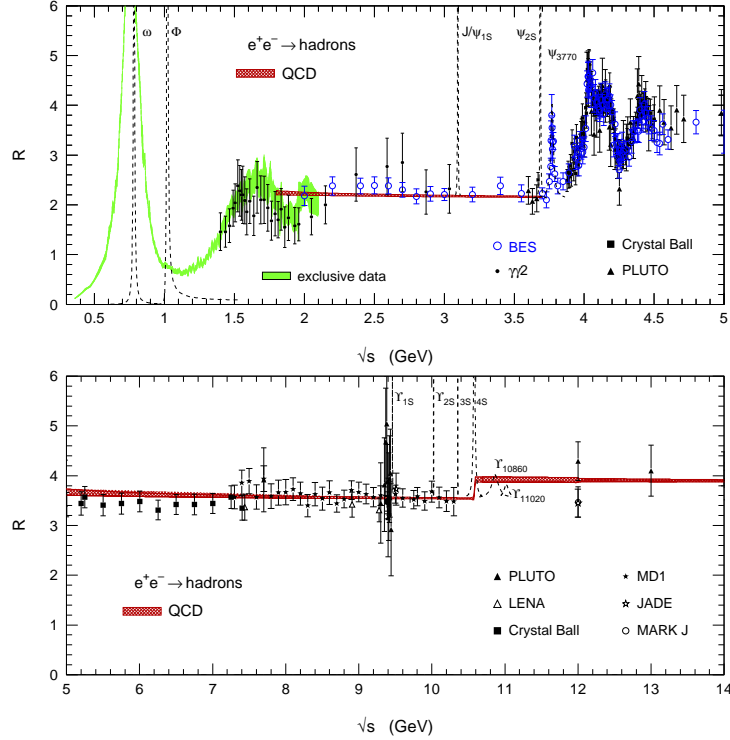


Figure 7. The R values from data and QCD used in the dispersion integral. The shaded band below 2.1 GeV represents the sum of the exclusive channels, not yet including the new BABAR results. Narrow resonances are indicated as dashed lines. All data points shown correspond to inclusive measurements. The cross-hatched band gives the prediction from QCD used in the calculation from 1.8 to 3.7 GeV and above 5 GeV.

the first time in recent years, the accuracy of the prediction exceeds that of the experiment. One should not forget however that the theoretical error is completely propagated from the systematic uncertainties of the input experiments, the estimation of which we totally depend. Also the evaluation of the systematic uncertainty on the hadronic light-by-light contribution is more subject to caution.

The discrepancy between the 2π spectral functions obtained from τ decays and from e^+e^- annihilation is still unresolved: it affects both the overall normalization and the shape. A particularly important test of the relative normalization is obtained, when comparing the measured branching fraction for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ to its Stan-

dard Model prediction from the e^+e^- spectral function, corrected for small isospin-breaking effects. Here the discrepancy amounts to 4.5 standard deviations.

In view of this problem and the fact that both Novosibirsk experiments agree at the 1% level, the hadronic contribution is computed only with e^+e^- data, excluding for the moment KLOE until their disagreement with Novosibirsk is understood. The choice of using only two experimental inputs (CMD-2 and SND) among the four available (+KLOE and τ decays) is clearly not satisfactory and even debatable, but represents in our view the most reasonable solution, all arguments considered. In this scenario we find that the Standard Model prediction of a_μ differs from the ex-

Table 2

The contributions of different final states in specified energy ranges to a_μ^{had} , given in units of 10^{-10} . Major improvements since Ref. [12] are obtained for (1) the $\pi^+\pi^-$ channel from CMD-2 (here preliminary results are used; final data have been published [23, 24, 25] since) and SND [22], while KLOE results are not included (see text), and (2) the $2\pi^+2\pi^-$ and other exclusive channels from BABAR [26, 27, 28, 29]. The uncertainty for missing radiative corrections, labeled 'rad', only concerns our ad hoc treatment [13] applied to older experimental data.

Modes	Energy range (GeV)	a_μ^{had}
$\pi^+\pi^-$	$2m_\pi-0.5$	$55.6 \pm 0.8 \pm 0.1_{\text{rad}}$
$\pi^+\pi^-$	$0.5-1.8$	$449.0 \pm 3.0 \pm 0.9_{\text{rad}}$
$2\pi^+2\pi^-$	$2m_\pi-1.8$	$13.1 \pm 0.4 \pm 0.0_{\text{rad}}$
$\pi^+\pi^-2\pi^0$	$2m_\pi-1.8$	$16.8 \pm 1.3 \pm 0.2_{\text{rad}}$
ω	$0.3-0.81$	$38.0 \pm 1.0 \pm 0.3_{\text{rad}}$
ϕ	$1.0-1.055$	$35.7 \pm 0.8 \pm 0.2_{\text{rad}}$
other exclusive	$2m_\pi-1.8$	$24.3 \pm 1.3 \pm 0.2_{\text{rad}}$
$J/\psi, \psi(2S)$	—	$7.4 \pm 0.4 \pm 0.0_{\text{rad}}$
R(QCD)	$1.8-3.7$	$33.9 \pm 0.5_{\text{QCD}}$
R(data)	$3.7-5.0$	$7.2 \pm 0.3 \pm 0.0_{\text{rad}}$
R(QCD)	$5.0-\infty$	$9.9 \pm 0.2_{\text{QCD}}$
sum	$2m_\pi-\infty$	$690.8 \pm 3.9 \pm 1.9_{\text{rad}} \pm 0.7_{\text{QCD}}$

perimental value by 3.3 standard deviations.

We are looking forward to the forthcoming results on the two-pion spectral function from KLOE and BABAR using the $\pi\pi/\mu\mu$ ratio. Since in this way vacuum polarization cancels, these data will help to reduce the systematic uncertainty due to the corrective treatment of radiative effects, always problematic in previous experiments normalized by luminosity. More data from BABAR on multihadron final states are also expected soon. With new experimental input to the vacuum polarization integrals, the quality of the prediction will improve, opening the way to a more precise direct determination of a_μ [47]. Unfortunately, some recent prospective work in the US [48] does not cast a bright future in this direction.

Acknowledgements

I would like to thank Andreas Höcker, Simon Eidelman and Zhiqing Zhang for our fruitful collaboration.

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